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Superluminal neutrino and spontaneous breaking of Lorentz invariance

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Generally speaking, the existence of a superluminal neutrino can be attributed either to re-entrant Lorentz violation at ultralow energy from intrinsic Lorentz violation at ultrahigh energy or to spontaneous breaking of fundamental Lorentz invariance (possibly by the formation of a fermionic condensate). Re-entrant Lorentz violation in the neutrino sector has been discussed elsewhere. Here, the focus is on mechanisms of spontaneous symmetry breaking.

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It is possible that OPERA's claimed discovery [1] of a superluminal muon-type neutrino does not come from the violation of Lorentz invariance but from unknown factors in the clock-synchronization process [2] or from a purely statistical effect [3]. In fact, it has been shown [4] that OPERA's claimed value $(v_{\nu_\mu} - c)/c \sim 10^{-5}$ is ruled out by the expected but unobserved energy losses from electron-positron-pair emission ($\nu_\mu \to \nu_\mu + e^- + e^+$), at least, as long as there exists a preferred frame from the Lorentz violation.

Still, the claim by OPERA has provided new impetus for the discussion on the possible sources of Lorentz violation. In order to engage in this discussion, let us assume that OPERA's result is correct qualitatively (existence of a superluminal muon-neutrino) even if not quantitatively (most likely, $|v_{\nu_\mu} - c|/c \ll 10^{-5}$).

Condensed-matter physics, which possesses an analog of Lorentz invariance (LI), now suggests several different scenarios of Lorentz violation (LV). Among them are:

(1a) LI is not a fundamental symmetry but an approximate symmetry which emerges at low energies and is violated at ultrahigh energies (cf. [5]).

(1b) Intrinsic LV at ultrahigh energies gives an emergent Lorentz-invariant theory at lower energies but ultimately, at or below an ultralow energy scale, induces a re-entrant violation of LI (see, e.g., Sec. 12.4 of [6]).

(2) LI is fundamental but broken spontaneously (see, e.g., [7, 8] and references therein).

In this Letter, we discuss the spontaneous breaking of Lorentz invariance (SBLI), that is, the spontaneous appearance of a preferred frame in the vacuum, which can be derived from Lorentz-invariant physical laws. The order parameter of SBLI can be a vector field b^α (for example, the vector field of Fermi-point splitting [9, 10] or an aether-type velocity field [11]), an emergent tetrad-type field e^α_a [12, 13, 14, 15], or any other field which is covariant but not invariant under Lorentz transformations.

If SBLI occurs only in the neutrino sector, which interacts weakly with the charged-matter sector, then SBLI has no direct impact on this other matter (certain indirect quantum-loop effects can be suppressed by near-zero mixing angles). The non-neutrino matter essentially does not feel the existence of the preferred reference frame. In fact, it is very well possible that SBLI occurs only for the neutrino field, because the other fermions have already experienced electroweak symme-

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try breaking phase transitions and are too heavy for any further type of symmetry breaking.

In condensed-matter physics, the re-entrant violation of LI, as well as Fermi-point splitting (FPS), follow from general topological properties of the vacuum (ground state) in 3–momentum space. Condensed matter provides many examples of homogeneous vacuum states, which have nontrivial topology in 3–momentum space [6]. Among them is a class of vacua which have Fermi points, exceptional points in 3–momentum space where the energy of fermionic excitations is nullified. Such a Fermi point (alternatively called Dirac or Weyl point) has a topological invariant. The existence of the Fermi point is thus protected by topology, or by the combined action of topology and symmetry. The Fermi point is robust to small perturbations of the system. In turn, different Fermi points may collide, annihilate, and split again, but their total topological charge is conserved. In this respect, Fermi points in 3–momentum space behave as topologically-charged 't Hooft–Polyakov magnetic monopoles in real space. The splitting or recombination of Fermi points represents a topological quantum phase transition. This type of quantum phase transition takes place, for example, in graphene, graphite, etc. (see, e.g., Figs. 4 and 6 in [16] for the splitting of a degenerate Fermi point into 3 and 4 elementary points, respectively).

Figure 8 in the review [10] (which elaborates on the discussion of the original research paper [9]) illustrates the special role of neutrinos. From the momentum-space topology of Fermi points, it follows that the phase transition away from the symmetric vacuum of the Standard Model with massless fermions may occur in two ways: either coinciding Fermi points with opposite topological charge annihilate each other, giving rise to a Dirac mass (the process commonly known as the Higgs mechanism), or coinciding Fermi points do not annihilate but split in momentum space, giving rise to Lorentz violation [9]. As mentioned above, it is possible that this FPS process only occurs for neutrinos, since all other particles have already obtained Dirac masses via the Higgs mechanism. After this splitting, the energy spectra of the left- and right-handed neutrino are given by the following expressions (the small neutrino mass can be neglected for the conditions relevant to the OPERA experiment):

$$g^{\alpha\beta}\left(c\,p_\alpha - \widetilde{b}^{\,\mathrm{L}}_\alpha\right)\left(c\,p_\beta - \widetilde{b}^{\,\mathrm{L}}_\beta\right) = 0\,, \tag{1a}$$

$$g^{\alpha\beta}\left(c\,p_\alpha - \widetilde{b}^{\,\mathrm{R}}_\alpha\right)\left(c\,p_\beta - \widetilde{b}^{\,\mathrm{R}}_\beta\right) = 0\,, \tag{1b}$$

with c the velocity of light *in vacuo*. In (1), we have put a tilde on the dimensional vector $\widetilde{b}_\alpha$ in order to distinguish it from the dimensionless vector b_α appearing below and we allow for $\widetilde{b}^{\,\mathrm{L}}_\alpha \neq \widetilde{b}^{\,\mathrm{R}}_\alpha$.

The possible role of FPS for the (qualitative) OPERA result has already been discussed in [17]. Here, we consider two scenarios for the spontaneous formation of a preferred reference frame. The first scenario corresponds to the appearance of a dimensionless 4–vector $(b^\alpha) = (b^0, \mathbf{b})$ in the neutrino vacuum. This 4–vector b^α interacts with the neutrino Dirac field in a way which does not violate the fundamental laws of special relativity:

$$S = \int d^3x\, c\, dt\; \overline{\psi}\, b^\alpha \left(-i\,\nabla_\alpha\right)\psi\,. \tag{2}$$

This action term corresponds to a momentum-dependent mass term,[2)] $M = p_\alpha\, b^\alpha/c$, which modifies the spectrum of the neutrino as follows:

$$p_\alpha\, p^\alpha \equiv E^2 - c^2\,|\mathbf{p}|^2 = \left(c\,p_\alpha\, b^\alpha\right)^2. \tag{3}$$

Let us, for example, take b^α to be a timelike vector, having $(b^0)^2 - |\mathbf{b}|^2 > 0$, and consider the particular reference frame with $\mathbf{b} = 0$. Assume $|b^0| < 1$. Then, the neutrino energy spectrum becomes

$$E^2\left[1 - (b^0)^2\right] = c^2\,|\mathbf{p}|^2\,, \tag{4}$$

which is superluminal for $b^0 \neq 0$. The same happens for a spacelike vector b^α. In the reference frame with $b^0 = 0$, the neutrino energy spectrum is given by

$$E^2 = c^2\,|\mathbf{p}|^2 + c^2\,(\mathbf{b}\cdot\mathbf{p})^2\,, \tag{5}$$

2) At this stage, it is clear that the same procedure can be followed with a Majorana mass term $\overline{\psi}{}^{\,c}_L\, m\, \psi_L$ in the action density, simply replacing the Majorana mass m by $-i\, b^\alpha\, \nabla_\alpha/c$.

which is both anisotropic and superluminal for $\mathbf{b} \neq 0$.

The 4–vector field b^α may emerge as the order parameter of the neutrino condensate,

$$b^\alpha \propto g^{\alpha\beta} \left\langle \overline{\psi} \, (-i \, \nabla_\beta) \, \psi \right\rangle \,, \tag{6}$$

in a theory with 4–fermion or multi-fermion interactions of the following type:

$$S_{\rm int} = \int d^3x \, c \, dt \;\, f(X) \quad , \tag{7a}$$

$$X = -g^{\alpha\beta} \left(\overline{\psi} \, \nabla_\alpha \, \psi \right) \left(\overline{\psi} \, \nabla_\beta \, \psi \right) , \tag{7b}$$

with appropriate dimensional constants entering the function f. This scenario gives a possible realization of the phenomenological Coleman–Glashow model [18] in terms of fermionic condensates [7, 8].

The second scenario involves another type of neutrino condensate, which also leads to SBLI. Specifically, this neutrino condensate gives rise to a tetrad-like field [12, 13, 14, 15]:

$$e^a_\alpha \propto \left\langle \overline{\psi} \, \gamma^a (-i \, \nabla_\alpha) \, \psi \right\rangle \,. \tag{8}$$

The induced tetrad field e^a_α must be added to the original fundamental tetrad $E_a^{(0)\alpha} = \mathrm{diag}(-1,\, c,\, c,\, c)$, and the fermionic action becomes

$$S = \int d^3x \, c \, dt \;\, E^\alpha_a \, \overline{\psi} \, \gamma^a (-i \, \nabla_\alpha) \, \psi \quad , \tag{9a}$$

$$E^\alpha_a = E_a^{(0)\alpha} + e^\alpha_a \,. \tag{9b}$$

Using the induced tetrad field $(e^\alpha_a) = \mathrm{diag}(b_0,\, 0,\, 0,\, 0)$ as an example, one obtains, for $0 < b_0 < 1$, the superluminal neutrino velocity $v_\nu = c/(1 - b_0)$.

The tetrad-type neutrino condensate (8) may provide a realization of the hypothetical spin-2 field discussed in [19]. A recent paper [20] presents another model, where a scalar-field composite plays a similar role as our condensate e^α_a. Also related may be a geometric model [21], based on a particular class of Finsler-spacetime backgrounds, which essentially modifies the effective metric entering particle dispersion relations.

This completes our discussion of two possible scenarios of spontaneous symmetry breaking to explain a superluminal neutrino. Spontaneous breaking of Lorentz invariance in the neutrino sector corresponds to the appearance of a preferred frame for the relevant neutrino: LI is violated if the neutrino momentum p_α is transformed but not the vacuum field b^α (or e^a_α) which is kept at a fixed value. Still, LI remains an exact symmetry of the physical laws: the invariance holds if both excitations and vacuum are transformed, that is, if the Lorentz transformation acts simultaneously on p_α and b^α (or e^a_α).

In these SBLI scenarios, as well as in the FPS scenario [17], the vacuum remains homogeneous, which is the reason why conservation of energy and momentum is exact. But the energy spectrum of the neutrino is modified, which must certainly have consequences for reactions involving neutrinos. Hence, there must be experimental constraints on b^α or e^a_α. Alternatively, the study of neutrino-interaction processes may provide valuable information on mechanisms proposed to explain non-standard (e.g., superluminal) propagation properties of the neutrinos.

The advantage of the spontaneous-symmetry-breaking scenario is that it stays fully within the realm of standard physics, which obeys special relativity. The multi-fermion interaction (7) can, in principle, originate from trans-Planckian physics, but we now have bounds [22, 23] indicating that Lorentz invariance holds far above the Planck energy scale, i.e., $E_{\rm LV} \gg E_{\rm Planck}$. This suggests that, if a neutrino has superluminal motion, it can be attributed either to re-entrant Lorentz violation at ultralow energy due to intrinsic (built-in) Lorentz violation at ultrahigh trans-Planckian energies [presumably with a re-entrance energy of order $(E_{\rm Planck}/E_{\rm LV})^n \, E_{\rm Planck}$ for $n \geq 1$] or to spontaneous breaking of fundamental Lorentz invariance [possibly by the formation of a fermionic condensate].

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